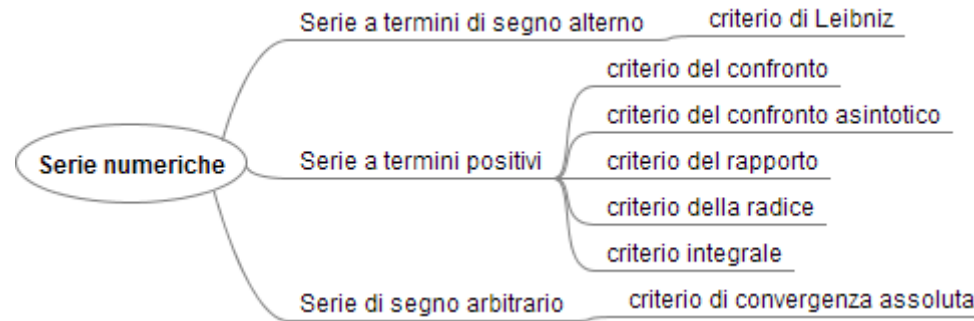


Serie numeriche



serie geometrica	serie di Mengoli
$\sum_{k=0}^{+\infty} q^k \begin{cases} \text{indeterminata} & \text{se } q \leq -1 \\ \text{converge a } \frac{1}{1-q} & \text{se } q < 1 \\ \text{diverge a } +\infty & \text{se } q \geq 1 \end{cases}$	$\sum_{k=1}^{+\infty} \frac{1}{k(k+1)} = 1$
serie armonica generalizzata	serie armonica generalizzata di segno alterno
$\sum_{k=1}^{+\infty} \frac{1}{k^\alpha} \begin{cases} \text{diverge} & \text{se } 0 < \alpha \leq 1 \\ \text{converge} & \text{se } \alpha > 1 \end{cases}$	$\sum_{k=1}^{+\infty} (-1)^k \frac{1}{k^\alpha} \begin{cases} \text{non converge} & \text{se } \alpha < 0 \\ \text{indeterminata} & \text{se } \alpha = 0 \\ \text{converge} & \text{se } \alpha > 0 \end{cases}$

Serie numeriche a termini positivi

criterio del confronto	criterio del confronto asintotico
$a_k \leq b_k, \forall k > k_0 \left\{ \begin{array}{l} \sum_{k=0}^{+\infty} b_k \text{ converge} \Rightarrow \sum_{k=0}^{+\infty} a_k \text{ converge} \\ \sum_{k=0}^{+\infty} a_k \text{ diverge} \Rightarrow \sum_{k=0}^{+\infty} b_k \text{ diverge} \end{array} \right.$	$\exists \lim_k \frac{a_k}{b_k} \in \mathbb{R} - \{0\} \left\{ \begin{array}{l} \sum_{k=0}^{+\infty} a_k \text{ converge} \Leftrightarrow \sum_{k=0}^{+\infty} b_k \text{ converge} \\ \sum_{k=0}^{+\infty} a_k \text{ diverge} \Leftrightarrow \sum_{k=0}^{+\infty} b_k \text{ diverge} \end{array} \right.$
criterio del rapporto	criterio della radice
$\exists \lim_k \frac{a_{k+1}}{a_k} \left\{ \begin{array}{l} \lim_k \frac{a_{k+1}}{a_k} < 1 \Rightarrow \sum_{k=0}^{+\infty} a_k \text{ converge} \\ \lim_k \frac{a_{k+1}}{a_k} > 1 \Rightarrow \sum_{k=0}^{+\infty} a_k \text{ diverge} \end{array} \right.$	$\exists \lim_k \sqrt[k]{a_k} \left\{ \begin{array}{l} \lim_k \sqrt[k]{a_k} < 1 \Rightarrow \sum_{k=0}^{+\infty} a_k \text{ converge} \\ \lim_k \sqrt[k]{a_k} > 1 \Rightarrow \sum_{k=0}^{+\infty} a_k \text{ diverge} \end{array} \right.$
criterio integrale (o di Maclaurin)	
$\exists \lim_k a_k \neq 0 \Rightarrow \sum_{k=0}^{+\infty} a_k \text{ diverge}$	$a_k = f(k), \forall k \geq k_0 \wedge f \text{ positiva, decrescente e continua in } [k_0, +\infty)$ $\left\{ \begin{array}{l} \int_{k_0}^{+\infty} f(x) dx \text{ converge} \Leftrightarrow \sum_{k=k_0}^{+\infty} a_k \text{ converge} \\ \int_{k_0}^{+\infty} f(x) dx \text{ diverge} \Leftrightarrow \sum_{k=k_0}^{+\infty} a_k \text{ diverge} \end{array} \right.$

Convergenza delle successioni di funzioni

convergenza puntuale	convergenza uniforme	
$\{f_n\}$ converge a f puntualmente in $A \Leftrightarrow \lim_n f_n - f = 0, \forall x \in A$ $\forall x \in A, \forall \varepsilon > 0 \rightarrow \exists \bar{n} = \bar{n}(\varepsilon, x): \forall n \geq n_0, n > \bar{n} \Rightarrow f_n - f < \varepsilon$	$\{f_n\}$ converge a f uniformemente in $A \Leftrightarrow \lim_n \sup_{x \in A} f_n - f = 0$ $\forall \varepsilon > 0 \rightarrow \exists \bar{n} = \bar{n}(\varepsilon): \forall n \geq n_0, n > \bar{n} \Rightarrow \sup_{x \in A} f_n - f < \varepsilon$ $\forall x \in A, \forall \varepsilon > 0 \rightarrow \exists \bar{n} = \bar{n}(\varepsilon): \forall n \geq n_0, n > \bar{n} \Rightarrow f_n - f < \varepsilon$	
Proprietà converg. uniforme	passaggio al limite sotto segno di integrale	passaggio al limite sotto segno di derivata
$\left\{ \begin{array}{l} f_n \text{ continue} \\ \{f_n\} \text{ unif. conv. a } f \end{array} \right. \Rightarrow f \text{ continua}$	$\left\{ \begin{array}{l} f_n \text{ integrabili} \\ \{f_n\} \text{ unif. conv. a } f \end{array} \right. \Rightarrow \left\{ \begin{array}{l} f \text{ integrabile} \\ \lim_n \int f_n dx = \int \lim_n f_n dx = \int f dx \end{array} \right.$	$\left\{ \begin{array}{l} f_n \in \mathcal{C}^1 \\ \{f_n\} \text{ punt. conv. a } f \\ \{f'_n\} \text{ unif. conv. a } g \end{array} \right. \Rightarrow \left\{ \begin{array}{l} f \in \mathcal{C}^1 \\ \{f_n\} \text{ unif. conv. a } f \\ f' = \left(\lim_n f_n \right)' = \lim_n f'_n = g \end{array} \right.$

Convergenza delle serie di funzioni

convergenza puntuale	convergenza uniforme	convergenza assoluta
$\sum_{k=k_0}^{+\infty} f_k$ converge a s (puntualmente) in $A \Leftrightarrow \{s_n\}$ converge a s puntualmente in A	$\sum_{k=k_0}^{+\infty} f_k$ converge a s uniformemente in $A \Leftrightarrow \{s_n\}$ converge a s uniformemente in A	$\sum_{k=k_0}^{+\infty} f_k$ converge assolutamente in $A \Leftrightarrow \sum_{k=k_0}^{+\infty} f_k $ converge $\forall x \in A$
Proprietà convergenza uniforme	integrazione per serie (serie integrabile termine a termine)	
$\left\{ \begin{array}{l} f_k \text{ continue} \\ \sum_{k=k_0}^{+\infty} f_k \text{ unif. conv. a } s \end{array} \right. \Rightarrow s \text{ continua}$	$\left\{ \begin{array}{l} f_k \text{ integrabili} \\ \sum_{k=k_0}^{+\infty} f_k \text{ unif. conv. a } s \end{array} \right. \Rightarrow \left\{ \begin{array}{l} s \text{ integrabile} \\ \sum_{k=k_0}^{+\infty} \int f_k dx = \int \sum_{k=k_0}^{+\infty} f_k dx = \int s dx \end{array} \right.$	
derivazione per serie (serie derivabile termine a termine)		
$\left\{ \begin{array}{l} f_k \in \mathcal{C}^1 \\ \sum_{k=k_0}^{+\infty} f_k \text{ punt. conv. a } s \\ \sum_{k=k_0}^{+\infty} f'_k \text{ unif. conv. a } g \end{array} \right. \Rightarrow \left\{ \begin{array}{l} s \in \mathcal{C}^1 \\ \sum_{k=k_0}^{+\infty} f_k \text{ unif. conv. a } s \\ s' = \left(\sum_{k=k_0}^{+\infty} f_k \right)' = \sum_{k=k_0}^{+\infty} f'_k = g \end{array} \right.$		

Serie di potenze

	serie geometrica	serie logaritmica	serie esponenziale	serie binomiale
$\sum_{k=1}^{+\infty} k^k x^k$ $A = \{0\}$ $R = 0$	$\sum_{k=0}^{+\infty} x^k$ $A = (-1,1)$ $R = 1$	$\sum_{k=1}^{+\infty} \frac{x^k}{k}$ $A = [-1,1)$ $R = 1$	$\sum_{k=0}^{+\infty} \frac{x^k}{k!}$ $A = \mathbb{R}$ $R = +\infty$	$\sum_{k=0}^{+\infty} \binom{\alpha}{k} x^k$ $A = (-1,1)$ $R = 1$

criterio del rapporto	criterio della radice
$\exists \lim_k \frac{ a_{k+1} }{ a_k } \begin{cases} \lim_k \frac{ a_{k+1} }{ a_k } = 0 & \Rightarrow R = +\infty \\ \lim_k \frac{ a_{k+1} }{ a_k } = l \in \mathbb{R}^+ & \Rightarrow R = \frac{1}{l} \\ \lim_k \frac{ a_{k+1} }{ a_k } = +\infty & \Rightarrow R = 0 \end{cases}$	$\exists \lim_k \sqrt[k]{ a_k } \begin{cases} \lim_k \sqrt[k]{ a_k } = 0 & \Rightarrow R = +\infty \\ \lim_k \sqrt[k]{ a_k } = l \in \mathbb{R}^+ & \Rightarrow R = \frac{1}{l} \\ \lim_k \sqrt[k]{ a_k } = +\infty & \Rightarrow R = 0 \end{cases}$

serie somma	serie prodotto (alla Cauchy)	serie derivata	serie integrale
$\begin{cases} R_1 = R_2 = R \Rightarrow R_{12} \geq R \\ R_1 \neq R_2 \Rightarrow R_{12} = \min\{R_1, R_2\} \end{cases}$	$R_{12} \geq \min\{R_1, R_2\}$	$R_{derivata} = R$	$R_{integrale} = R$

Sviluppi di Maclaurin

$$T(0) = \sum_{k=0}^{+\infty} \frac{f^{(k)}(0)}{k!} x^k$$

$(1+x)^\alpha = \sum_{k=0}^{+\infty} \binom{\alpha}{k} x^k \quad A = \begin{cases} (-1,1) & \alpha \leq -1 \\ (-1,1] & -1 \leq \alpha < 0 \\ [-1,1] & \alpha > 0 \end{cases} \quad R = 1$	$\frac{1}{1+x} = \sum_{k=0}^{+\infty} (-1)^k x^k \quad A = (-1,1) \quad R = 1$
$\frac{1}{1-x} = \sum_{k=0}^{+\infty} x^k \quad A = (-1,1) \quad R = 1$	$\ln(1+x) = \sum_{k=1}^{+\infty} (-1)^{k-1} \frac{x^k}{k} \quad A = (-1,1] \quad R = 1$
$e^x = \sum_{k=0}^{+\infty} \frac{x^k}{k!} \quad A = \mathbb{R} \quad R = +\infty$	$\operatorname{arctg} x = \sum_{k=0}^{+\infty} (-1)^k \frac{x^{2k+1}}{2k+1} \quad A = (-1,1] \quad R = 1$
$\cosh x = \sum_{k=0}^{+\infty} \frac{x^{2k}}{(2k)!} \quad A = \mathbb{R} \quad R = +\infty$	$\cos x = \sum_{k=0}^{+\infty} (-1)^k \frac{x^{2k}}{(2k)!} \quad A = \mathbb{R} \quad R = +\infty$
$\operatorname{senh} x = \sum_{k=0}^{+\infty} \frac{x^{2k+1}}{(2k+1)!} \quad A = \mathbb{R} \quad R = +\infty$	$\operatorname{sen} x = \sum_{k=0}^{+\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!} \quad A = \mathbb{R} \quad R = +\infty$

Funzioni non esprimibili elementarmente

funzione degli errori

$$\operatorname{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{+\infty} \frac{(-1)^k}{k!} \int_0^x t^{2k} dt$$

funzione seno integrale

$$\operatorname{Si}(x) = \int_0^x \frac{\operatorname{sen} t}{t} dt = \sum_{k=0}^{+\infty} \frac{(-1)^k}{(2k+1)!} \int_0^x t^{2k} dt$$

funzione seno integrale di Fresnel

$$S(x) = \int_0^x \operatorname{sen} \left(\frac{\pi}{2} t^2 \right) dt = \sum_{k=0}^{+\infty} \frac{(-1)^k}{(2k+1)!} \left(\frac{\pi}{2} \right)^{2k+1} \int_0^x t^{4k+2} dt$$

funzione coseno integrale di Fresnel

$$C(x) = \int_0^x \cos \left(\frac{\pi}{2} t^2 \right) dt = \sum_{k=0}^{+\infty} \frac{(-1)^k}{(2k)!} \left(\frac{\pi}{2} \right)^{2k} \int_0^x t^{4k} dt$$

Spazi vettoriali	$\mathbb{R}^n = \{\vec{v} = (x_1 \dots x_n) x_1, \dots, x_n \in \mathbb{R}\}$	$\tilde{\mathcal{C}}_{2\pi} = \{f: \mathbb{R} \rightarrow \mathbb{R} f \text{ continua a tratti, periodica di periodo } 2\pi, \text{ regolarizzata}\}$
prodotto scalare	$\vec{v}_A \cdot \vec{v}_B = \sum_{k=1}^n x_{A_k} x_{B_k}$	$(f, g) = \int_0^{2\pi} f(x)g(x)dx$ (prodotto scalare integrale)
norma	$\ \vec{v}\ = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{\sum_{k=1}^n x_k^2}$	$\ f\ _2 = \sqrt{(f, f)} = \sqrt{\int_0^{2\pi} [f(x)]^2 dx}$ (norma quadratica)
distanza	$d(\vec{v}_A, \vec{v}_B) = \vec{v}_B - \vec{v}_A = \sqrt{\sum_{k=1}^n (x_{A_k} - x_{B_k})^2}$	$d(f, g) = \ f - g\ _2 = \sqrt{\int_0^{2\pi} [f(x) - g(x)]^2 dx}$ (distanza quadratica)
base ortogonale	$\mathcal{B}_{\mathbb{R}^n} = \{k_1 \vec{e}_1, \dots, k_n \vec{e}_n\}$	$\mathcal{F} = \{\varphi_0(x); \varphi_1(x), \dots; \psi_1(x), \dots\}$ $\varphi_0(x) = 1 \quad \varphi_k(x) = \cos(kx) \quad \psi_k(x) = \text{sen}(kx)$
base ortonormale	$\mathcal{B}_{\mathbb{R}^n}^{\mathcal{C}} = \{\vec{e}_1, \dots, \vec{e}_n\}$	$\hat{\mathcal{F}} = \{\hat{\varphi}_0(x); \hat{\varphi}_1(x), \dots; \hat{\psi}_1(x), \dots\}$ $\hat{\varphi}_0(x) = \frac{1}{\sqrt{2\pi}} \quad \hat{\varphi}_k(x) = \frac{1}{\sqrt{\pi}} \cos(kx) \quad \hat{\psi}_k(x) = \frac{1}{\sqrt{\pi}} \text{sen}(kx)$
sottospazio vettoriale	$\vec{v} = \sum_{k=1}^3 x_k \vec{e}_k = x\vec{e}_1 + y\vec{e}_2 + z\vec{e}_3 \in \mathbb{R}^3$ $x = \vec{v} \cdot \vec{e}_1 \quad y = \vec{v} \cdot \vec{e}_2 \quad z = \vec{v} \cdot \vec{e}_3$	$p_n = a_0 \hat{\varphi}_0(x) + \sum_{k=1}^n [a_k \hat{\varphi}_k(x) + b_k \hat{\psi}_k(x)] \in \mathcal{P}_n$ $a_0 = (p_n, \hat{\varphi}_0(x)) \quad a_k = (p_n, \hat{\varphi}_k(x)) \quad b_k = (p_n, \hat{\psi}_k(x))$

Serie di Fourier (di funzioni periodiche di periodo $T > 0$)

$$f(x) \approx \sum_{k=0}^{+\infty} a_k \varphi_k + \sum_{k=1}^{+\infty} b_k \psi_k = a_0 + \sum_{k=1}^{+\infty} \left[a_k \cos\left(k \frac{2\pi}{T} x\right) + b_k \operatorname{sen}\left(k \frac{2\pi}{T} x\right) \right]$$

$$a_0 = \frac{1}{T} (f, \varphi_0) = \frac{1}{T} \int_0^T f(x) dx$$

$$a_k = \frac{2}{T} (f, \varphi_k) = \frac{2}{T} \int_0^T f(x) \cos\left(k \frac{2\pi}{T} x\right) dx$$

$$b_k = \frac{2}{T} (f, \psi_k) = \frac{2}{T} \int_0^T f(x) \operatorname{sen}\left(k \frac{2\pi}{T} x\right) dx$$

$f(x)$ pari

$$f(x) \approx \sum_{k=0}^{+\infty} a_k \varphi_k = a_0 + \sum_{k=1}^{+\infty} a_k \cos\left(k \frac{2\pi}{T} x\right)$$

$$a_0 = \frac{2}{T} \int_0^{T/2} f(x) dx$$

$$a_k = \frac{4}{T} \int_0^{T/2} f(x) \cos\left(k \frac{2\pi}{T} x\right) dx$$

$f(x)$ dispari

$$f(x) \approx \sum_{k=1}^{+\infty} b_k \psi_k = \sum_{k=1}^{+\infty} b_k \operatorname{sen}\left(k \frac{2\pi}{T} x\right)$$

$$b_k = \frac{4}{T} \int_0^{T/2} f(x) \operatorname{sen}\left(k \frac{2\pi}{T} x\right) dx$$

$$\sum_{k=1}^{+\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$$

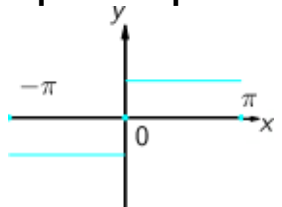
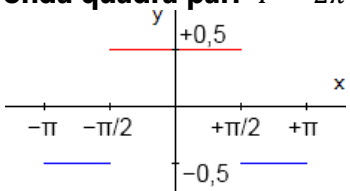
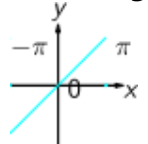
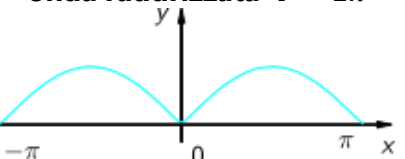
$$\sum_{k=1}^{+\infty} \frac{1}{k^4} = \frac{\pi^4}{90}$$



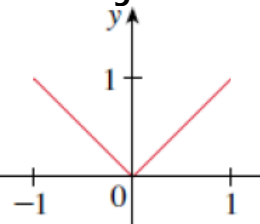
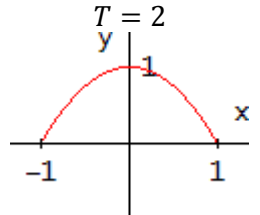
$$\sum_{k=1}^{+\infty} \frac{1}{4k^2 - 1} = \frac{1}{2}$$

$$\sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{k^2} = \frac{\pi^2}{12}$$

$$\sum_{k=0}^{+\infty} \frac{1}{(2k+1)^4} = \frac{\pi^4}{96}$$

$$\sum_{k=0}^{+\infty} \frac{(-1)^k}{2k+1} = \frac{\pi}{4}$$

Onde I	Espressione della funzione	Serie di Fourier	Serie numeriche
<p>onda quadra dispari $T = 2\pi$</p> 	$f(x) = \begin{cases} -\frac{1}{2} & \text{se } -\pi < x < 0 \\ +\frac{1}{2} & \text{se } 0 < x < \pi \end{cases}$	$f(x) \approx \frac{2}{\pi} \sum_{k=0}^{+\infty} \frac{1}{2k+1} \text{sen}[(2k+1)x]$	$\sum_{k=0}^{+\infty} \frac{(-1)^k}{2k+1} = \frac{\pi}{4}$
<p>onda quadra pari $T = 2\pi$</p> 	$f(x) = \begin{cases} -\frac{1}{2} & \text{se } \frac{\pi}{2} < x < \pi \\ +\frac{1}{2} & \text{se } x < \frac{\pi}{2} \end{cases}$	$f(x) \approx \frac{2}{\pi} \sum_{k=1}^{+\infty} \frac{\text{sen}\left(\frac{k\pi}{2}\right)}{k} \cos(kx)$	
<p>onda a dente di sega $T = 2\pi$</p> 	$f(x) = x \quad \text{se } -\pi < x < \pi$	$f(x) \approx 2 \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{k} \text{sen}(kx)$	$\sum_{k=1}^{+\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$
<p>onda raddrizzata $T = 2\pi$</p> 	$f(x) = \text{sen } x $	$f(x) \approx \frac{2}{\pi} - \frac{4}{\pi} \sum_{k=1}^{+\infty} \frac{1}{4k^2 - 1} \cos(2kx)$	$\sum_{k=1}^{+\infty} \frac{1}{4k^2 - 1} = \frac{1}{2}$

Onde II	Espressione della funzione	Serie di Fourier	Serie numeriche
<p>onda parabolica $T = 2\pi$</p> 	$f(x) = x^2 \quad \text{se } -\pi < x < \pi$	$f(x) \approx \frac{\pi^2}{3} + 4 \sum_{k=1}^{+\infty} \frac{(-1)^k}{k^2} \cos(kx)$	$\sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{k^2} = \frac{\pi^2}{12}$ $\sum_{k=1}^{+\infty} \frac{1}{k^4} = \frac{\pi^4}{90}$
<p>onda cubica $T = 2\pi$</p> 	$f(x) = x^3 \quad \text{se } -\pi < x < \pi$	$f(x) \approx 2 \sum_{k=1}^{+\infty} (-1)^k \frac{6 - \pi^2 k^2}{k^3} \text{sen}(kx)$	
<p>onda triangolare $T = 2$</p> 	$f(x) = x \quad \text{se } -1 < x < 1$	$f(x) \approx \frac{1}{2} - \frac{4}{\pi^2} \sum_{k=1}^{+\infty} \frac{1}{(2k-1)^2} \cos[(2k-1)\pi x]$	$\sum_{k=1}^{+\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$ $\sum_{k=0}^{+\infty} \frac{1}{(2k+1)^4} = \frac{\pi^4}{96}$
<p>$T = 2$</p> 	$f(x) = 1 - x^2 \quad \text{se } -1 < x < 1$	$f(x) = \frac{2}{3} + \frac{4}{\pi^2} \sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{k^2} \cos(k\pi x)$	$\sum_{k=1}^{+\infty} \frac{1}{k^4} = \frac{\pi^4}{90}$ $\sum_{k=1}^{+\infty} \frac{(-1)^{k+1}}{k^2} = \frac{\pi^2}{12}$

